https://www.linkedin.com/feed/update/urn:li:activity:6528831234183958528
The determinant

$$\det\begin{pmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{pmatrix}$$ is equal to zero for all values of α, if:

(a) a,b,c are in AP; (b) a,b,c are in GP; (c) a,b,c are in HP; (d) non of these.

Solution by Arkady Alt , **San Jose** ,**California**, **USA**.

$$\det\begin{pmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{pmatrix} = \det\begin{pmatrix} a & b & 0 \\ b & c & 0 \\ a\alpha+b & b\alpha+c & -(a\alpha+b)\alpha-(b\alpha+c) \end{pmatrix} =$$

$(ac-b^2)(-(a\alpha+b)\alpha-(b\alpha+c)) = (b^2-ac)(c+2b\alpha+a\alpha^2)$.

Since $(b^2-ac)(c+2b\alpha+a\alpha^2) = 0$ for all values of α only if $b^2-ac = 0$ (because otherwise $c+2b\alpha+a\alpha^2 = 0$ for any $\alpha \in \mathbb{R} \Leftrightarrow a = b = c = 0$).

So, answer is (b).